

# Optimum Jack Loads for Static and Fatigue Tests

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**An automated procedure used at the Northrop Grumman Corporation to determine the jack loads for static and fatigue structural tests is described. The procedure includes a recently developed method to automatically compute jack loads that satisfy jack-capacity and jack-direction constraints, while providing a best fit to known internal moment, shear, and torsion loads. The engineering steps required to solve this problem have been automated and integrated with the optimization procedure. Compared with the semimanual approaches generally used in the industry, the new method has saved one-third to one-half of the engineering time required to determine the jack loads on the A-6E and C-2A aircraft fatigue tests while, providing a better fit to the moment, shear, and torsion loads.**

## Introduction

**I**N structural static and fatigue tests, hydraulic jack loads are applied to reproduce, with as much fidelity as possible, the loads to which the structure will be subjected in service. This paper describes a general method to determine the jack loads. Naval aircraft are used as specific examples. All of the significant loading conditions, such as catapult takeoffs, flight maneuvers, free and arrested landings, dynamic landings, gravity loads while at rest on the ground, etc., must be applied. These loading conditions produce very different loading patterns that are applied with one test configuration. Loading conditions are reproduced by matching the moment, shear, and torsion resultants (MVTs) along a reference axis (usually the elastic axis). In general, all six load components at prespecified points on the reference axis are matched.

Our previous procedure employed best-fit regression to minimize the sum of the squares of the errors between the desired and the test MVTs. This method did not automatically modify jack loads that exceeded their physical limitations. For example, maximum loads are limited either because of jack capacity, load-introduction capability, e.g., at some points, only tension or compression loads can be applied, or airframe structural limits. When the regression procedure produced a value of load for a jack that could not be applied because of physical constraints, the excess load was redistributed manually. This was very time-consuming when many of the conditions required for a fatigue test (500–1500) had to be adjusted manually. Moreover, the resulting solutions generally did not minimize the sum of the squares of the errors. An admittedly incomplete survey of industrial and government engineers responsible for the development of jack loads for structural testing indicates that, except for Kaman Aerospace Corp., Bloomfield, CT (as discussed later), similar, and often less sophisticated, methods are used to solve this problem.

In the new procedure, the problem is recast as a constrained minimization problem. By using this formulation, the physical constraints can be automatically accounted for, while ensuring

an exact minimum for the sum of the squares of the errors. This new procedure has been implemented in a computer program developed at Northrop Grumman called Optical (optimum for a constrained and linear system). The program generates jack loads that match the design MVT curves by minimizing the sum of the squares of the errors subject to the physical constraints on the jacks (such as jack capacity and/or jack direction). Several available numerical search procedures also were explored to solve this problem; however, because the current method computes an exact solution for the minimum, it always provided a minimum that was at least identical to, and often better than, the solution provided by the existing methods. Because this comparison was not exhaustive, a faster method could exist; however, because only seconds are required to compute a solution, the optimization is, by far, the fastest of the engineering steps employed in generating the jack loads. Consequently, a faster method would not appreciably affect the total time needed to generate jack loads. However, in the future more complex problems could arise with more jack loads, more loading conditions, more MVTs, types of constraints, and the possible need to match internal finite element loads. Consequently, additional research is recommended to thoroughly compare the method with other available approaches.

A thorough literature survey was conducted and revealed only one other published procedure to automatically determine the jack-load magnitudes, the generalized force-determination (GFD) procedure developed at Kaman Aerospace Corporation<sup>1</sup>. GFD uses experimental information to determine the coefficient matrix relating the test loads to the desired measurements. Information conditioning filters out the effects of measurement noise from this matrix. Regression is used to obtain a best fit to desired measurements, and if actuators exceed limiting values, they are reset at their limits and the loads for the other actuators are recalculated. This procedure is repeated iteratively until no actuators exceed their limits. However, unlike the approach in the present paper, the actuators that are set at their limiting values are fixed at these values in all subsequent iterations. If some of these actuators could vary in their allowable range during the iteration process a better fit would be achieved in many cases; consequently, GFD does not always produce an optimum fit to the desired measurements.

In contrast, in Optical, the coefficient matrix is determined analytically, and a method is used to identify which previously fixed actuators should vary during the iterations used to satisfy the load constraints. By allowing these actuators to vary, an exact optimum fit to the MVTs is achieved.

In addition, those fully loaded jacks that should be increased in capacity to most improve the fit to the MVTs are identified.

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The program has been incorporated into Cogs, a Northrop Grumman general finite element analysis computer system, so that data and results flow automatically between Optical and the other program modules such as the MVT calculation program and the plotting program. This procedure was used successfully for the calculation of the jack loads on the A-6E fuselage fatigue test and on the C-2 total-aircraft fatigue test. Numerous load conditions are employed in an aircraft fatigue test; for the A-6E fuselage fatigue test a total of 470 conditions were handled, whereas for the C-2A a total of 3147 conditions were matched. Consequently, computing the jack loads is a lengthy task that generally requires many person-months. By minimizing the time to complete the optimization step, the new procedure saved one-third to one-half of the person-months, while producing a better fit to the MVTs.

### Engineering Procedure

Many steps in the overall engineering procedure to determine jack loads are employed throughout the industry; however, the steps are repeated here for completeness. The steps involve the generation of desired MVTs, separation of loads into discrete and distributed quantities, assembly of geometry and capacities, generation of coefficient matrix, computation of optimum jack loads, and comparison of desired and applied MVTs.

The basic data that are required are 1) geometry and capacity of independent jacks; 2) applied load conditions; and 3) definition of the elastic axis for each aircraft component, along which the MVTs will be computed.

Data on the geometry and capacity of independent jacks include information on how the independent jacks are beamed or distributed onto the structure through whiffletrees, straps, or other fixtures. The basic data required to assemble the load conditions are defined by the different engineering disciplines. Concurrent loads are combined and each load condition is balanced by incorporating additional inertia loads, as required.

The following operations are carried out for each major component of the aircraft. First, the left and the right wings are processed, then the empennage, and finally the fuselage. The fuselage is processed last because inputs from both wings and the empennage are required for the fuselage.

1) Generate desired MVTs for fatigue conditions: Force and moment resultant MVTs ( $F_x$ ,  $F_y$ ,  $F_z$ ,  $M_x$ ,  $M_y$ , and  $M_z$ ), are generated along the elastic axis for all of the fatigue conditions using a program module in Cogs called MVT. This module requires input data that fully define each load condition. These loads are analytically derived, but usually are modified with input from test data, if they are available, such as air-load data from wind-tunnel tests or from actual flights. The analyses are performed with specific mathematical models that include 1) aerodynamic air loads models, 2) weights models, 3) vehicle maneuvers models, 4) dynamic models, and 5) landing-gear loads models.

Loads data that are generated with the various models must be combined, balanced, and applied to a common model. For example, for a landing, aerodynamic loads are generated with aerodynamic air-load models; inertia loads are generated with weights models; and ground loads are generated with a combination of landing-gear and transient-response dynamic models. All of the loads are transferred to the structural finite element model where they are combined and then balanced with inertia loads. These combined loads completely define the static or fatigue conditions and are used to generate the MVTs on the elastic axis of the particular component.

2) Separate loads into discrete and distributed loads: The loads are divided into two groups, discrete loads and distributed loads. Discrete loads are defined as loads coming from discrete components of the aircraft such as the engine and the landing gears. The intent is to apply these loads exactly with designated jacks. Distributed loads are those resulting from the inertia of the aircraft, distributed air loads, and any other ef-

fects that have not been accounted for. The distributed loads are applied with a different set of jacks whose values are determined by matching the MVT curves with a constrained minimization procedure.

All sources of discrete loads must be considered, such as 1) engine(s), 2) pylons, 3) nose landing gear, 4) main landing gears, 5) arresting hook, 6) rotodome, and 7) empennage.

At this point, the discrete loads may be modified because of practical limitations. For example, for catapult conditions the nose landing-gear loads obtained from analysis often are modified by the criteria report that specifies the maximum tow bar force that is to be applied. Also, to apply the discrete loads exactly with designated jacks, each discrete component would require six jacks. The allocation of six jacks for each discrete component is usually not feasible either because of the limitation on the total number of available jacks, limitations on physical space, or cost considerations. Experience and engineering judgment must be used in deciding which force and moment components will be matched and which will be approximated.

The full definition of the application of the discrete loads during the test includes the geometry (location and direction) and magnitude of all the jacks. When this is completed, MVTs are computed for the discrete jacks along the respective reference lines. These MVTs are then subtracted from the MVTs of the total loads that have been previously generated. The resulting MVTs are the values to be matched by the prescribed jacks and form the components of a vector called  $\{\bar{y}\}$ . These MVTs represent the distributed loads, any discrete loads not accounted for, or other load adjustments deemed necessary. The procedure is repeated for each loading condition so that there are as many vectors  $\{\bar{y}\}$  as there are loading conditions.

3) Assemble geometry and capacities of the jacks that will be used to simulate the distributed loads: The definition of the independent jacks that will be used to simulate the distributed loads consists of the following information for each jack: 1) location ( $X$ ,  $Y$ ,  $Z$  coordinates in the airplane reference system); 2) direction of applied load; 3) tension/compression capability; 4) method of application to the aircraft (whiffletrees, straps, other fixtures and associated geometry); and 5) capacity. This information is determined by the coordinated efforts of the test group (fixture considerations), the stress group (airplane-structure and load-capacity considerations), and the structural mechanics group (applied-load requirements).

4) Generate MVTs for unit values of the above jacks along the reference line of the component: Using the geometry of the jacks, values for the moments, shears, and torsion are calculated along the reference line of the component caused by unit values of the jacks. The MVTs for each jack form a column of a matrix called  $[\bar{A}]$ .

5) Perform constrained minimization for distributed jacks simulating distributed loads: The  $\{\bar{y}\}$  matrix, capacity and direction of each jack, and  $[\bar{A}]$  matrix are used as input to the Optical program. Optical then performs the constrained minimization analysis to obtain values of all the jacks that give a least-squares fit to the MVT curves but also adhere to the physical constraints of the test setup. Mathematically, Optical solves the problem of finding the jack loads (elements of  $\{x\}$ ) that provide the best-fit solution to the equation:

$$[Q](\bar{A}\{x\} - \{\bar{y}\}) \approx 0 \quad (1)$$

where  $[Q]$  is a diagonal matrix that is used to weight equations that are considered more important and must be satisfied more exactly. It is assumed that there are  $m$  MVTs and  $n$  jacks, and, as is generally the case, there are more MVTs than jacks; therefore,  $[\bar{A}]$  is an  $m \times n$  coefficient matrix with  $m \geq n$ . The approximately equal sign  $\approx$  is used because there are more equations than unknowns; consequently, all of the equations cannot generally be satisfied. Each jack load (component  $x_i$  of the solution  $\{x\}$ ) is constrained such that it must lie between

a prespecified lower bound  $x_{iL}$  and upper bound  $x_{iU}$ . The residual to Eq. (1) is

$$\{\mathbf{r}\} = [\mathbf{Q}][\tilde{\mathbf{A}}]\{\mathbf{x}\} - \{\bar{\mathbf{y}}\} \quad (2)$$

Optical finds the best fit to Eq. (1), subject to the constraints on the jack loads, in the sense that the 2-norm of the residual  $\|\{\mathbf{r}\}\| = \sqrt{\{\mathbf{r}\}^T\{\mathbf{r}\}}$  is minimized. In practice, solutions for a group of problems can be obtained in one submittal, where each problem can have a different load condition and constraints on the jacks.

The printed output includes the regression solution to Eq. (1), i.e., the solution if there were no constraints as well as the corresponding residual and its norm. These values may be compared with the constrained solution to determine the restriction on the results that is caused by imposing the constraints.

An activity-indicator vector  $\{\mathbf{w}\}$  also appears in the printed output. The elements  $w_i$  are nonzero only for solution components  $x_i$  that are at their upper or lower bounds. The size of  $w_i$  is an indication of the improvement in the solution that would be obtained by relaxing the corresponding constraint. For example, in structural test problems, increasing the capacity of jacks with large  $w_i$  would significantly improve the fit to the desired MTVs.

The derivation of the procedure used in Optical is presented in the next section. While iteration is used to satisfy the constraints, we have found that solutions can be obtained very quickly. Typically, a group of 300 problems with different load conditions, 30 jacks, and 1000 MVT sampling points is solved in less than 2 min on an IBM RS6000 workstation.

6) Combine discrete and distributed jacks: For each condition, the values of the set of jacks simulating the discrete loads obtained in step 2 are combined with the values of the set of jacks simulating the distributed loads obtained in step 5 to give the values of the complete set of jacks for that condition.

7) Plot test MVTs vs design MVTs: Using the geometry of the set of jacks and their values from step 6 as input to the MVT command in the COGS system, moment, shear, and torsion curves are computed for the test conditions. These curves are then compared to the MVT curves generated in step 1.

### Analytical Method

Equations (1) and (2) are rewritten as

$$[\mathbf{A}]\{\mathbf{x}\} \approx \{\mathbf{y}\} \quad \text{and} \quad [\mathbf{A}]\{\mathbf{x}\} - \{\mathbf{y}\} = \{\mathbf{r}\} \quad (3)$$

where

$$[\mathbf{A}] = [\mathbf{Q}][\tilde{\mathbf{A}}] \quad \text{and} \quad \{\mathbf{y}\} = [\mathbf{Q}]\{\bar{\mathbf{y}}\} \quad (4)$$

The solution  $\{\mathbf{x}\}$  minimizes  $\|\{\mathbf{r}\}\|^2 = \|[\mathbf{A}]\{\mathbf{x}\} - \{\mathbf{y}\}\|^2 = \sum_i q_i^2 (\sum_j \tilde{a}_{ij} x_j - \bar{y}_i)^2$ , where  $q_i$  is the  $i$ th diagonal element of  $[\mathbf{Q}]$ ,  $\tilde{a}_{ij}$  is the  $ij$ th element of  $[\tilde{\mathbf{A}}]$ , and  $\bar{y}_i$  is the  $i$ th element of  $\{\bar{\mathbf{y}}\}$ . Usually, most of the  $q_i$  are set equal to 1; however, if  $q_i$  is given a high value, the solution  $\{\mathbf{x}\}$  will provide a better fit to the  $i$ th equation  $\sum_j \tilde{a}_{ij} x_j - \bar{y}_i \approx 0$ , at the expense of the fit to the other equations, because small errors in the  $i$ th equation are multiplied by  $q_i^2$  and can therefore still have a significant effect on  $\|[\mathbf{A}]\{\mathbf{x}\} - \{\mathbf{y}\}\|^2$ .

Iteration is employed to satisfy the constraints. The iterations start with the regression solution, i.e., the solution that is obtained by disregarding the constraints, then two types of iteration are employed. Initially, an aggressive set of iterations, called basic iterations, is employed to rapidly move the solution toward the optimum, minimum  $\|\{\mathbf{r}\}\|$ , value. However, the basic iterations can cause the solution to oscillate, thereby preventing convergence. Consequently, the basic iterations are followed by a conservative set of iterations, called supplementary iterations, that move the solution slowly toward the optimum, but very significantly improve convergence.

During any iteration, some of the elements of  $\{\mathbf{x}\}$ , called active variables, are permitted to vary; and others, called inactive variables, are fixed at limiting values in accordance with logic that will be described. Accordingly,  $\{\mathbf{x}\}$  is separated into  $\{\mathbf{x}_1\}$ , which contains the active variables (with zeroes in the locations of the inactive variables) and  $\{\mathbf{x}_2\}$ , which contains the inactive variables (with zeroes in the locations of the active variables).  $[\mathbf{A}]$  also is separated into  $[\mathbf{A}_1]$  and  $[\mathbf{A}_2]$ , where  $[\mathbf{A}_1]$  contains the columns corresponding to the active variables, and  $[\mathbf{A}_2]$  contains the columns corresponding to the inactive variables, and each matrix contains zeroes in the other columns. Accordingly, Eq. (3) is rewritten as

$$[\mathbf{A}_1]\{\mathbf{x}_1\} + [\mathbf{A}_2]\{\mathbf{x}_2\} \approx \{\mathbf{y}\} \quad (5)$$

or

$$[\mathbf{A}_1]\{\mathbf{x}_1\} \approx \{\mathbf{y}_1\} \quad (6)$$

where

$$\{\mathbf{y}_1\} = \{\mathbf{y}\} - [\mathbf{A}_2]\{\mathbf{x}_2\} \quad (7)$$

By setting  $[\mathbf{A}]\{\mathbf{x}\} = [\mathbf{A}_1]\{\mathbf{x}_1\} + [\mathbf{A}_2]\{\mathbf{x}_2\}$  in Eq. (3) and substituting Eq. (7)

$$\{\mathbf{r}\} = [\mathbf{A}_1]\{\mathbf{x}_1\} - \{\mathbf{y}_1\} \quad (8)$$

so that  $\{\mathbf{r}\}$  is the residual for Eq. (6) as well as Eq. (1) and (3). To minimize  $\|\{\mathbf{r}\}\|$ , the regression solution<sup>2</sup> is employed; i.e., the following equation is solved for  $\{\mathbf{x}_1\}$ :

$$([\mathbf{A}_1]^T[\mathbf{A}_1])\{\mathbf{x}_1\} = [\mathbf{A}_1]^T\{\mathbf{y}_1\} \quad (9)$$

However,  $\{\mathbf{x}_1\}$  may not be the final solution because some of the constraints may be violated. Also, solutions, satisfying the constraints, with lower  $\|\{\mathbf{r}\}\|$  could be obtained if some of the previously inactive variables were made active. An iteration procedure is employed to satisfy the constraints and drive the solution to the optimum.

### Basic Iterations

The general procedure for effecting the basic iterations is as follows. During the first iteration each variable is active and a regression solution is obtained. After each iteration each active variable that violated a constraint is reset to the constrained value and made inactive during the next iteration. Also, variables that were inactive during the previous iteration are examined to determine whether the solution might be improved, i.e., lower the value of  $\|\{\mathbf{r}\}\|$ , by making these variables active during the next iteration. All variables that might improve the solution are made active. Convergence is achieved when a solution is obtained that is identical to the solution for the previous iteration (in the sense that it has identical sets of active and inactive variables). An oscillation occurs when a solution is obtained that is identical to the solution for an iteration prior to the previous iteration because a group of iterations would continuously be repeated if the iterations were continued. The basic iteration procedure terminates when convergence is achieved, an oscillation is detected, or a prespecified number of basic iterations is completed.

To determine whether a previously inactive variable should be made active, each element of  $\{\mathbf{x}_2\}$  is varied by a differential amount  $\delta\{\mathbf{x}_2\}$  to determine how this would have changed the solution. If  $\{\mathbf{x}_2\}$  were varied,  $\{\mathbf{x}_1\}$  would change to  $\{\mathbf{x}_1\} + \delta\{\mathbf{x}_1\}$ . From Eq. (5)

$$[\mathbf{A}_1](\{\mathbf{x}_1\} + \delta\{\mathbf{x}_1\}) + [\mathbf{A}_2](\{\mathbf{x}_2\} + \delta\{\mathbf{x}_2\}) \approx \{\mathbf{y}\} \quad (10)$$

To minimize the norm of the residual, the following equation is solved for  $\{\mathbf{x}_1\} + \delta\{\mathbf{x}_1\}$ :

$$[\mathbf{A}_1]^T[\mathbf{A}_1](\{\mathbf{x}_1\} + \delta\{\mathbf{x}_1\}) = [\mathbf{A}_1]^T(\{\mathbf{y}\} - [\mathbf{A}_2](\{\mathbf{x}_2\} + \delta\{\mathbf{x}_2\}))$$

Using Eqs. (7) and (9)

$$\delta\{\mathbf{x}_1\} = -([A_1]^T[A_1])^{-1}[A_1]^T[A_2]\{\mathbf{x}_2\} \quad (11)$$

The square of the norm of the error residual to Eq. (10) is

$$E + \delta E = \|[A_1](\{\mathbf{x}_1\} + \delta\{\mathbf{x}_1\}) + [A_2](\{\mathbf{x}_2\} + \delta\{\mathbf{x}_2\}) - \{\mathbf{y}\}\|^2$$

Using Eqs. (7) and (8)

$$E + \delta E = \|\{\mathbf{r}\} + [A_1]\delta\{\mathbf{x}_1\} + [A_2]\delta\{\mathbf{x}_2\}\|^2$$

If Eq. (11) is substituted and products of small quantities are neglected

$$\delta E = -2\{\mathbf{w}\}^T\delta\{\mathbf{x}_2\} \quad (12)$$

where  $E = \{\mathbf{r}\}^T\{\mathbf{r}\}$  and

$$\{\mathbf{w}\} = -[A_2]^T([I] - [A_1]([A_1]^T[A_1])^{-1}[A_1]^T)\{\mathbf{r}\}$$

From Eqs. (8) and (9),  $[A_1]^T\{\mathbf{r}\} = 0$ ; therefore

$$\{\mathbf{w}\} = -[A_2]^T\{\mathbf{r}\} \quad (13)$$

The inactive variables that lower  $E$  when permitted to vary by a differential amount into the feasible region, i.e., into the region where the constraints are not violated, can now be identified. From Eq. (12), it is seen that if inactive variable  $i$  is at its lower bound, it will lower  $E$  when increased by a differential amount only if the corresponding element  $w_i$  of  $\{\mathbf{w}\}$  is positive. Conversely, if inactive variable  $i$  is at its upper bound, it will lower  $E$  when decreased by a differential amount only if the corresponding element  $w_i$  of  $\{\mathbf{w}\}$  is negative. The variables that satisfy these criteria are made active during the next iteration because they have the potential to improve the solution. This is accomplished by inserting them into  $\{\mathbf{x}_1\}$  and inserting a zero into the corresponding element of  $\{\mathbf{x}_2\}$ . The  $w_i$  are called the activity indicators and are sometimes referred to as Lagrange multipliers. The magnitude of  $w_i$  is a measure of the improvement in the solution that can be obtained by activating the corresponding variable.

Every active variable that violated a constraint is reset to its constrained value and is made an inactive variable for the next iteration. To account for resetting these variables, the reset value is placed into  $\{\mathbf{x}_2\}$ , and a zero is inserted into the corresponding element of  $\{\mathbf{x}_1\}$ . The next iteration is then initiated by separating  $[A]$  into  $[A_1]$  and  $[A_2]$ , computing  $\{\mathbf{y}_1\}$  from Eq. (7), and  $\{\mathbf{x}_1\}$  from Eq. (9). Then, the procedure is repeated until convergence, an oscillation, or a maximum number of basic iterations has been completed.

#### Supplementary Iterations

The basic iterations quickly move the solution from the initial unconstrained regression solution to a vector that is much closer to the final converged solution that satisfies the constraints. However, this procedure often does not converge; therefore, after conducting a few basic iterations, the slower, but more likely to converge, supplementary-iteration procedure described in this subsection is performed. The supplementary-iteration procedure is similar to the basic-iteration procedure, except that the solution is moved more slowly toward the optimum. In each iteration, instead of changing the status of all variables that violate the constraints from active to inactive, the status of only one variable is changed. Also, instead of changing the status of all inactive variables that would improve the solution to active, generally only the status of one variable is changed.

As indicated in Fig. 1, often several constraints are violated by the solution  $\{\mathbf{x}\}$  that is obtained at the beginning of any

iteration. Instead of moving the solution all the way from the previous solution  $\{\mathbf{x}\}^{(p)}$  to the new solution  $\{\mathbf{x}\}$ , which is in the infeasible region, a move of  $\{\mathbf{x}\} - \{\mathbf{x}\}^{(p)}$ , it is moved only part of the way to the constraint boundary, a move of  $\mu(\{\mathbf{x}\} - \{\mathbf{x}\}^{(p)})$ , where  $\mu$  is determined such that the only constraint violated is the first one encountered. The corresponding variable is made inactive

$$\{\mathbf{x}\}^* = \{\mathbf{x}\}^{(p)} + \mu(\{\mathbf{x}\} - \{\mathbf{x}\}^{(p)}) \quad (14)$$

Because  $\|\{\mathbf{r}\}\|^2$  is a quadratic function of  $\{\mathbf{x}\}$ , movement of the solution from  $\{\mathbf{x}\}^{(p)}$  toward the partially unconstrained optimum  $\{\mathbf{x}\}$  guarantees a lower value of  $\|\{\mathbf{r}\}\|$ . Therefore, during the supplementary iterations,  $\|\{\mathbf{r}\}\|$  cannot increase. It either decreases, or it could remain fixed during any iteration where one, or more, constraints are immediately violated.

To implement this process, Eq. (14) is solved for the value of  $\mu$  that would move the solution to the constraint boundary for each element  $x_i$  of  $\{\mathbf{x}\}$  that violates a constraint. The result  $\mu_i$  has the following value

$$\mu_i = \begin{cases} \frac{x_{iL} - x_i^{(p)}}{x_i - x_i^{(p)}} & \text{if } x_i < x_{iL} \\ \frac{x_{iU} - x_i^{(p)}}{x_i - x_i^{(p)}} & \text{if } x_i > x_{iU} \end{cases} \quad (15)$$

To move the solution to the first constraint boundary that is encountered, the value of  $\mu$  in Eq. (14) is set to the smallest of the  $\mu_i$ .

To determine whether activating any inactive design variable would improve the solution, the sign of  $w_i$  of each design variable is examined in accordance with the procedure of the preceding subsection. If the sign indicates that an improvement would be achieved then this variable is considered a candidate to be made active. In accordance with Eq. (12), the magnitude of  $w_i$  is a measure of the improvement in the solution that can be obtained by activating the corresponding variable; therefore, the variable that is activated is the candidate with the largest  $|w_i|$ .

The iterations are repeated where, at most, one variable is made inactive, and, at most, one variable is made active during each iteration. An exception occurs when more than one candidate has the same value of the largest  $|w_i|$ . In this case all of the candidates with the same maximum  $|w_i|$  are made active.

#### Numerical Example

For the A-6E fuselage fatigue test, a total of 470 load conditions were matched. As an example, the maximum normal load condition, a 6.5-g symmetric flight maneuver, is discussed. Matches, i.e., comparisons of desired and achieved MVTs, of vertical shear (Fig. 2), vertical bending moment (Fig. 3), and torsion (Fig. 4) are shown for the wing; whereas for

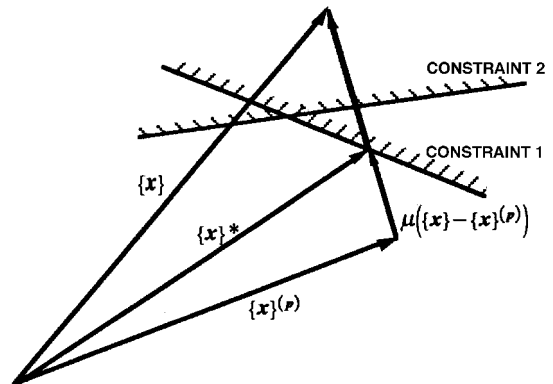


Fig. 1 Movement of the solution to the constraint boundary.

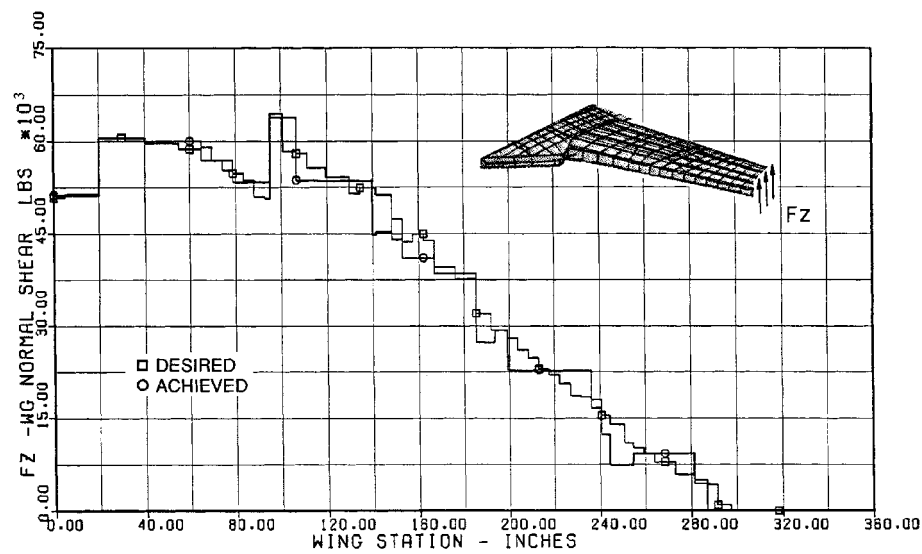


Fig. 2 Desired and achieved vertical shear (FZ) on left wing for 6.5-g symmetric flight condition of A-6E fuselage fatigue test.

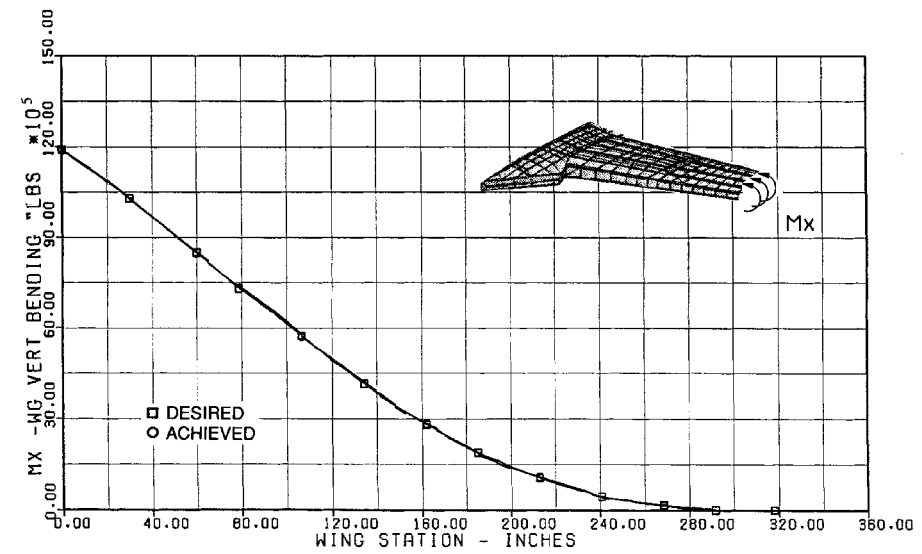


Fig. 3 Desired and achieved vertical bending moment (MX) on left wing for 6.5-g symmetric flight condition of A-6E fuselage fatigue test.

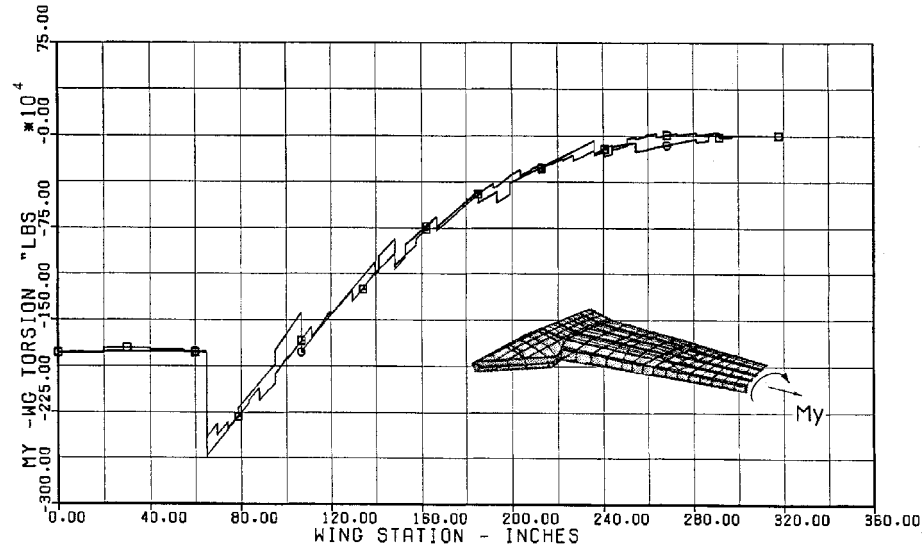


Fig. 4 Desired and achieved torsion load (MY) on left wing for 6.5-g symmetric flight condition of A-6E fuselage fatigue test.

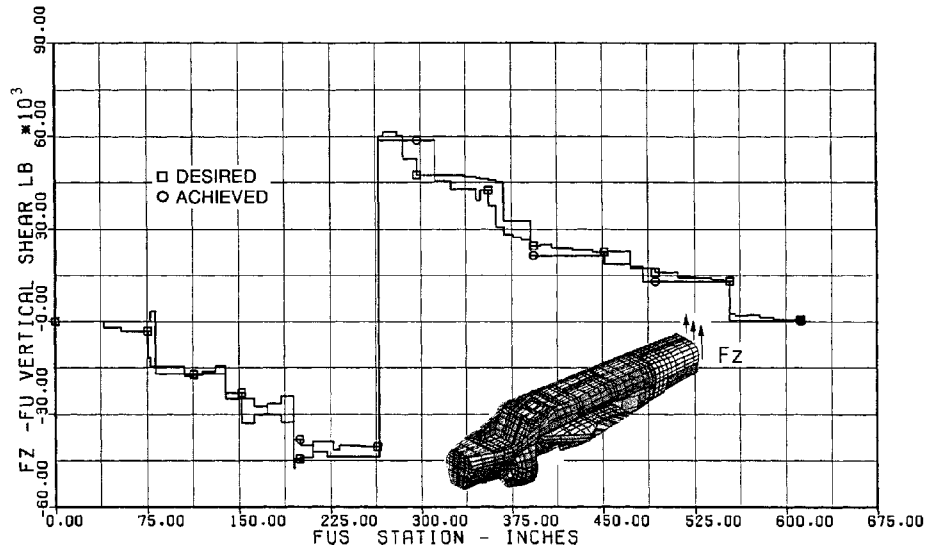


Fig. 5 Desired and achieved vertical shear (FZ) on fuselage for 6.5-g symmetric flight condition of A-6E fuselage fatigue test.

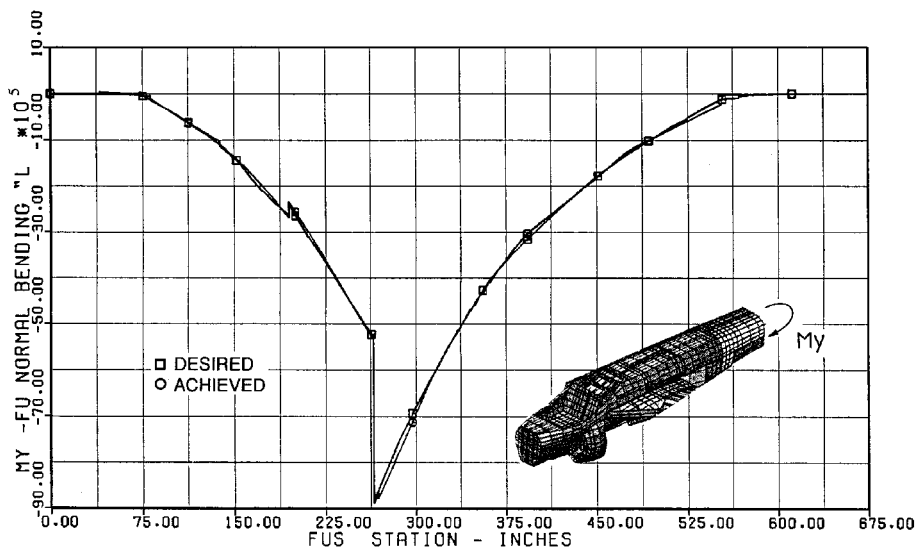


Fig. 6 Desired and achieved normal bending moment (MY) on fuselage for 6.5-g symmetric flight condition of A-6E fuselage fatigue test.

the fuselage matches of vertical shear (Fig. 5) and normal bending moment (Fig. 6) are illustrated. Theoretically, the procedure provides an exact optimum solution. Note that the matches between the desired analysis and test values of the MVTs all are excellent, while the constraints of the jacks are not violated. For the A-6E, a total of 120 independent jacks were used, 35 for each wing and 50 for the fuselage.

### Conclusions

The determination of jack loads for static and fatigue aircraft tests is a lengthy and expensive task. An engineering procedure is described that incorporates a new method to rapidly generate jack loads and provide an exact optimal match to desired MVTs. On the A-6E fuselage fatigue test and the C-2A total-

aircraft fatigue test it is estimated that the procedure saved one-third to one-half of the person-months while producing a better fit to the MVTs. For each solution, the optimization portion of the procedure requires only a fraction of a second on an IBM RS6000 work station; however, additional research is required to determine whether other optimization approaches would solve the problem more rapidly.

### References

- <sup>1</sup>Tomashofski, C. A., Nagy, E. J., and Keary, P. E., "An Improved Method of Structural Dynamic Test Design for Ground Flying and Its Application to the SH-2F and SH-2G Helicopters," *Flight Testing*, CP-519, AGARD, 1992, pp. 28-1-28-23.
- <sup>2</sup>Jennings, A., *Matrix Computations for Engineers and Scientists*, Wiley, New York, 1977.